Symmetry of STRUCTURE

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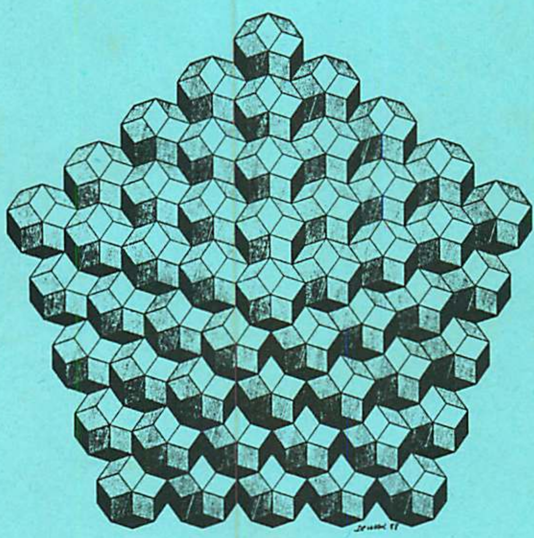
Abstracts

I.



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SOME ASPECTS OF SYMMETRY IN SCHOOL EDUCATION, OPTIMAL CONTROL AND RELATIVITY

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In this report three aspects of symmetry are considered. These aspects are united together not only on account of interests of the author, but in consequence of their organic connection too.

1. The question on a part of symmetry in a school education is extremely actual now. The attention to this aspect of symmetry was attracted, first of all, by F. KLEIN and H.WEIL. Since the time of EUCLID the proofs in school teaching of geometry are based on consideration of "chains" of equal (more precisely, congruent) triangles. This method of praving makes reasonings to be logically strong. However this method represents a blind alley, which is not connected with the present and the future of the science and does not give ways into other branches of the science and into applications. A sharp criticism of "triangular" proofs in school geometry was given by a very known French mathematician J.DIEUDONNE. Nowhere, except school and entrance examinations to colleges, this method does not apply.

On the contrary, the ideas of symmetry are directed to future. They provide a deep connection between school geometry, other branches of mathematics and applications. The question is not only to apply the reflection, the central symmetry and other geometrical transformations in school, but in general to use a group approach for comprehending of geometrical facts. The set of all geometrical transformations, which keep distances between points and map a given figure onto itself, is named to be the symmetry group of the figure. A knowledge of symmetry group of a figure determined its geometrical properties. The symmetry group of a parallelogram contains a central symmetry, and all the properties of this figure are followed from this fact. The existence of two reflections in the symmetry group of a rhombus implies additional its properties. All the properties of regular polygons are determined by their symmetry groups etc. (Fig. 1).

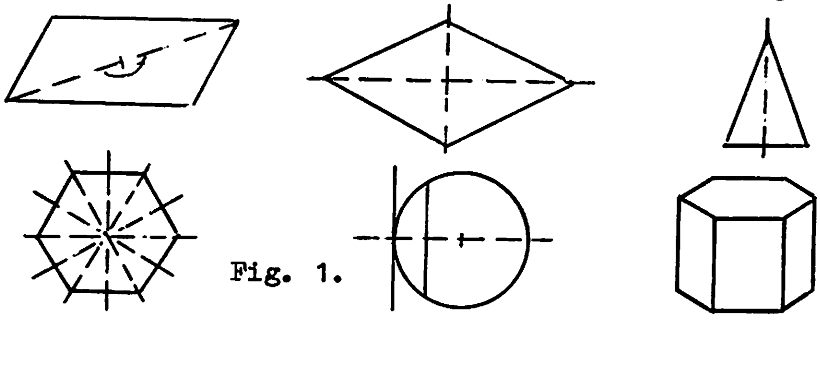

Fig. 1.

In essential, the whole variety of the facts of the school geometry is a manifestation of the Symmetry.

The group approach is very important for nuclear physics, partial relativity theory, crystallography and other sciences. That is why a rebuilding of school geometrical teaching on the foundation of symmetry aspects (instead of an archaic method of "triangular" proofs) is a very actual problem of modern school education.

2. The mathematical theory of optimal control is an important achievement of mathematics in XX century. The maximum principle in many its versions is the central result of this theory. It was advanced (as a hypothesis) by L.PONTRJAGIN. A proof of the maximum principle in linear case was given by R.GAMKRELIDZE and in general, nonlinear case it was given by the author of this report. A paramount role in this theory (both in statements and in proofs) belongs to ideas of symmetry. There is a duality of HAMILTON's type in the statement of the maximum principle, which is expressed by a symmetry of formulas relatively phase coordinates and auxiliary variables.

This symmetry has an essential importance for theoretical reasonings and numerical solutions of different optimal control problems. In the most frequent case an optimal control problem has a symmetric structure. It means that the control region, which describes a set of admissible contrôls, is centrally symmetric (relatively the origin) and the equations of motion of an object are symmetric relatively the origin of the phase space. As a consequence of the symmetric structure of an optimal control problem we obtain a symmetric picture of the BELLMAN's sphere.

The symmetry of the statement of the maximum principle and a symmetric character of the phase portrait of the system of optimal trajectories are a display of a deep symmetry, which is contained in a proof of the maximum principle. This proof is based on using of separation theory of convex cones, which as a matter of fact is deeply symmetric and generalizes symmetry principles of elementary geometry. Just the geometrical separation theory of convex cones was used by the author of report for developing of a "tent method". This method is now the most effective one of solution of optimisation tasks and other extremal problems. There are some ideas of symmetry in its basis too.

3. We have already mentioned, that the partial relativity theory is an original four-dimensional time-space geometry, which is based upon the LORENTZ group as the fundamental symmetry group. There are very interesting and rich in content aspects of symmetry in the general relativity theory too. These aspects are closely associated with the optimal control theory. An intention consists in using a postulate of "displacement" of a light sphere. This postulate allows to deduce the following

four-dimensional time-space metric:

$$ds^2=\left(c^2-\frac{2Gm}{r}\right)dx^0dx^0+\frac{2\sqrt{2Gm}}{r^{3/2}}x_p dx^p dx^0+h_{pq}dx^p dx^q. \qquad (ж)$$

From the metric (ж) it is possible by a very simple manner to obtain the SCHWARZSCHILD's metric, which is very known in relativity. This way twice uses ideas of symmetry. First, the gravitation field of a resting mass possesses a spherical symmetry, and this fact is used essentially for deducing of the metric (ж). Second, the passage from (ж) to SCHWARZSCHILD's metric is based upon a symmetrizing too. This symmetrizing consists in a passage to the "location" metric, for which the values of the light velocities in two opposite directions are equally. Even the fact, that the SCHWARZSCHILD's metric can be obtained by such a simple way, is unlikely an accidental coincidence. It is very likely, that the "displacement" postulate describes a mechanism of an interaction of a particle with gravitation field. Perhaps, this mechanism will allow to obtain physical foundations of the general relativity theory (which must differ from EINSTEIN's, pure geometrical one). Let us note, that the metric (ж) gives the same good coincidence with experiments as the SCHWARZSCHILD's one, since the equations of geodesic lines are in both the metrics the same.

Fig.2 shows the sense of the "displacement" postulate.

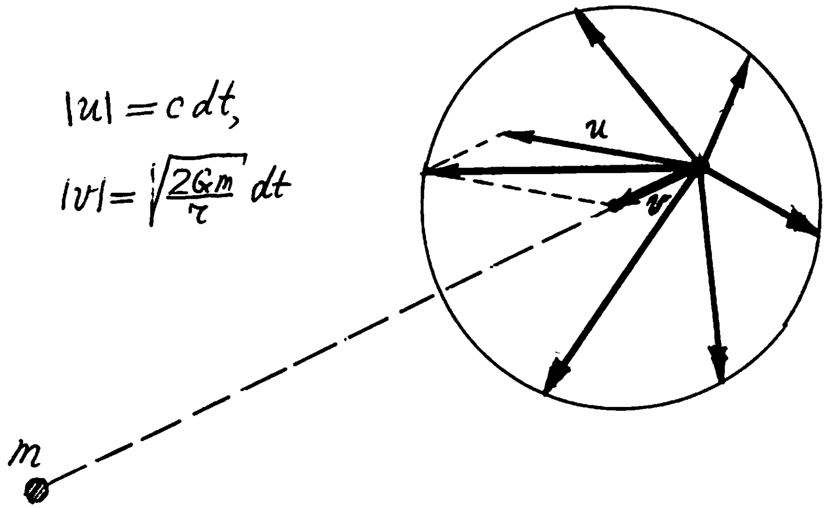

Fig. 2

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