Symmetry

of

STRUCTURE

an interdisciplinary Symposium
Abstracts
I.

Edited by Gy. Darvas and D. Nagy

BUDA
Pesta
August 13-19, 1989
HUNGARY
NONTRIVIAL SYMMETRY PROPERTIES OF THE NONLINEAR
BOLTZMANN EQUATION

A.V. Bobylev

In the classical kinetic theory the state of the gas at the
time \( t \) is characterized by the distribution function \( f(\chi, \nu, t) \)
of its molecules with respect to the coordinates \( \chi \in \mathbb{R}^3 \) and
the velocities \( \nu \in \mathbb{R}^3 \). The time evolution of this function
is described by the Boltzmann equation

\[
\frac{\partial f}{\partial t} + \nu \cdot \frac{\partial f}{\partial \chi} = I(f, f),
\]

where \( I(f, f) \) is the nonlinear collision integral.

The report is devoted to the review of the mathematical re-
sults connected with the group properties of this equation. It
is the integro-differential equation, that's why it is difficult
to use the standart methods of the group analyses. Obviously we
can find the set of the symmetry transformations for the Boltzmann
equation which are connected with the shifts, the rotations and
the scale transformations of the independent variables and the
function \( f \). We call here these transformations (with the Galilean
group) the trivial transformations because these properties are
well-known and don't give us any new information about the Boltz-
mann equation and its solutions.

We have also at least two types of the nontrivial transfor-
mations for certain intermolecular potentials. Firstly it is
the Lee-Backlund group for the potential \( U(\zeta) = \alpha \zeta / \zeta^5 \)
(Maxwell gas) in the spatially uniform case [1]. Secondly it is
the Lee group of the projective transformations for the potential
\[ \bar{\mathcal{U}}(z) = \sqrt{\mathcal{U}^2(\mathcal{U})} \] in the spatially nonuniform case. We analyse in the report the consequences of these two classes of the transformations (the conservation laws, exact solutions) and their connection with the symmetry properties of the Euler and Navier-Stokes equations.

REFERENCES
