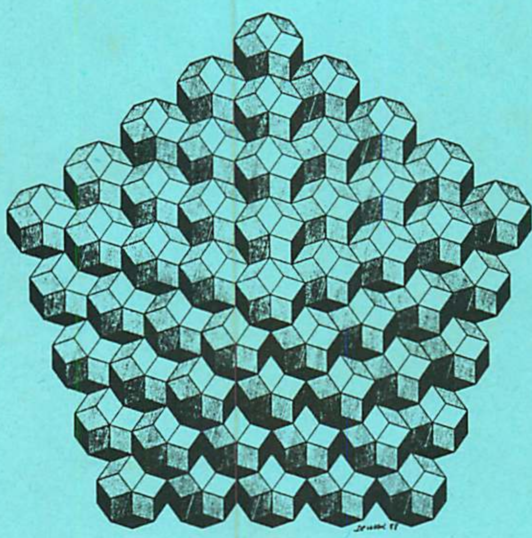
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda
pest

August 13-19, 1989

hungary

NONTRIVIAL SYMMETRY PROPERTIES OF THE NONLINEAR BOLTZMANN EQUATION

A.V. Bobylev

In the classical kinetic theory the state of the gas at the time t is characterized by the distribution function $f(x, v, t)$ of its molecules with respect to the coordinates $x \in \mathbb{R}^3$ and the velocities $v \in \mathbb{R}^3$. The time evolution of this function is described by the Boltzmann equation

$$\frac{\partial f}{\partial t} + v \cdot \frac{\partial f}{\partial x} = I(f, f),$$

where $I(f, f)$ is the nonlinear collision integral.

The report is devoted to the review of the mathematical results connected with the group properties of this equation. It is the integro-differential equation, that's why it is difficult to use the standart methods of the group analyses. Obviosly we can find the set of the symmetry transformations for the Boltzmann equation which are connected with the shifts, the rotations and the scale transformations of the independent variables and the function f. We call here these transformations (with the Galilei group) the trivial transformations because these properties are well-known and don't give us any new information about the Boltzmann equation and its solutions.

We have also at least two types of the nontrivial transformations for certain intermolecular potentials. Firstly it is the Lee-Backlund group for the potential $U(r) = \alpha / r^5$ (Maxwell gas) in the spatially uniform case [1]. Secondly it is the Lee group of the projective transformations for the potential

$U(r) = \alpha / r^2$ [?] in the spatially nonuniform case. We analyse in the report the consequences of these two classes of the transformations (the conservation laws, exact solutions) and their connection with the symmetry properties of the Euler and Navier--Stokes equations.

REFERENCES

1. Bobylev A.V. Exact and approximate methods in the theory of the nonlinear Boltzmann and Landau kinetic equations. - Moscow, Keldysh Inst. of Appl. Math., USSR Ac. of Sci., 1987.
2. Bobylev A.V., Ibragimov N.H. The connection of the symmetry properties of the equations in the dynamics, kinetic theory and hydrodynamics. - Math. modelling, v.I, N 3, pp.101-112, 1989.