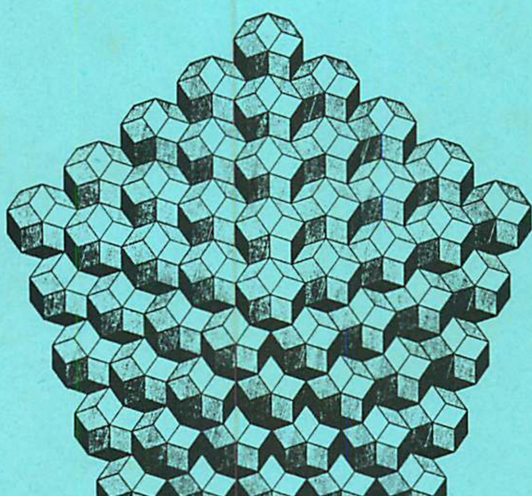
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda
pest

August 13-19, 1989

hungary

APPROXIMATE SYMMETRIES OF EQUATIONS WITH A SMALL PARAMETER

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Roughly, the investigation of different physical processes is made based on the simpliest model representations. They are chosen to truly describe a given process on the whole. When investigating such models and constructing their particular solutions, of great help are methods of group analysis (i.e. Lie and Lie-Backlund transformation groups). The consideration of subsequent approximations gives us a succession of models, each of them considering some additional factor. Though these factors are little, they can materially change the general picture of the process. As a rule taking into account the subsequent approximation narrows the allowed group of the simpliest model equation. It greatly limits the use of classical symmetries for the investigation of differential equations.

The introduction of approximate symmetries allows to overcome this problem and to construct the theory of stable relatively small perturbations of differential equations symmetries. The theory of approximate symmetries is based on a concept of the approximate transformation group, which is allowed (with a definite degree of accuracy) by an equation with a small parameter. Based on the analogue of Lie's theorem for approximate groups, an infinitesimal description of approximate single-parameter transformation groups is developed, and the determinative equations for the construction of approximate symmetries are isolated.

In general, the perturbated equation (the model taking into account the additional factors) admits not any symmetry of non-perturbated equation in the form of approximate symmetry. If such a succesion of symmetries is admitted, such symmetries will be called stable symmetries.

In the way of example we made a group classification of nonlinear wave equation (which for example describes iso-entropic motion of the lequid in a tube with a little dissipation in Lagrange coordinates). It is demonstrated, that not every symmetry of the wave equation is stable, i.e. not all of it's exact symmetries are succeded in the form of approximate symmetries of the wave equation, which takes into account the small dissipation. For convenience of comparison we calculated the exact symmetries of the wave equation with small parameter and isolated the cases when the use of approximate groups gives an additional expantion.

The whole group on non-perturbated equation is fully succeded by a perturbated equation (in the form of a group of approximate symmetries) only exceptionally. In case such a succession takes place with any degree of accuracy, one can introduce for consideration a new object - the formal symmetries. The change from approximate symmetries to formal ones allows us to get rid of the condition of parameter smallness and consider it as a calibrating ellement (i.e. the formal symmetries represent the formal power series). To stable symmetries belong the symmetries of transfer equation which are succeded by evolutionary equations with an arbitrary degree of accuracy. Among these one distiguishes such symmetries, which satisfy the condition of breaking the formal series. In case such a condition is fulfilled, we get known Lie-Backlund operators. The approach we propose gives a new method of constructing Lie-Backlund symmetries. Contrary to the usual method, here the process of calculating the coordinates of the canonic Lie-Backlund operator is directed not from the higher (by derivatives) numbes to lower, but vica versa. Such an approach allows also to find out how does it happen, that the groups, which are separately admitted by the equations of Burgers and Korteweg-de Vries, vanish (within the limits of the Lie-Backlund theory) when we consider the Burgers-Korteweg-de Vries equation. When we go over to the Burgers-Korteweg-de Vries equation it appeares, that the corresponding Lie-Backlund operators are transformed into the formal ones which don't satisfy the condition of breaking the formal series.

With the approximate (formal) symmetries closely connected are the approximate (formal) Backlund transformations. The stability of all symmetries of the transfer equation indicates the possibility of on approximate transformation (with any degree of accuracy) of an evolutionary equation into transfer equations. Such transformation is executed with the aid of the formal Backlund transformation. As far as the transfer equation is linearized by means of a point transformation, all evolutionary equations are formally linearized.